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Incomplete Blocks in Box Compounds and its agronomic applications

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Abstract

An alternative design for agricultural experimentation is presented which consisted on repetitions of the composed designs of Box using incomplete blocks. The system of incomplete blocks in repeated and non repeated designs is discussed, under the assumption that in the agronomic field such designs offer comparative advantages in the estimation of the variance of the experimental error, with respect to the designs without blocking. With such aim the structure and efficiency of the Composed Central Orthogonal design (C.C.O) in both repeated and non repeated is analyzed plus the Composed Central Rotary design (C.C.R.) in both repeated and non repeated, C.C.O. in incomplete blocks repeated and not repeated and C.C.R. in incomplete block repeated and not repeated. For such designs, the analyses of the variance and the estimations of the coefficients of the model were discussed and a practical example with data corresponding to maize fertilization (*Zea mays* L.). Finally, the designs in incomplete blocks with those in complete blocks are compared, concluding that it is advisable and favorable the alternative of composed designs of Box in repeated incomplete blocks, under the conditions of the study.

Key words: Repeated Incomplete Blocks, Comparison of designs, Rotary Central Compound design, Orthogonal Central Compound design, relative efficiency, experimental error variance, regression coefficients variance.

Introduction

In the industrial field the response surface methodology has an important role on the obtaining of optimum operation conditions. The response surface methodology emerges as an alternative to solve the necessities of surpassing the inconvenient that appear in the designs with plot arrangement of treatments, when the number of factors and level grow considerably. Chacín (4) says on this matter, that the main disadvantage of the complete factorial arrangements consist on increasing the number of factors of levels of these, the number of

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treatments also increases substantially, making impossible the conduction of experiments in the field.

Box and Wilson (1) use the composed designs to solve experiment problems that involve a number of factors and levels which are relatively high, mainly under conditions that allow an adequate control of the experimental error, however, it is well known that on the agriculture research, specifically on the field experimentation, the variability of the experimental material is high and the experimental error can not always be estimated satisfactorily as on the industrial processes, therefore, these tend to be high. Nevertheless, several efforts have been made allowing that the response surface design would compete favorably when pretending to obtain a model that would study more efficiently the surface to be fixed.

For the classic response surface designs in processes where the experimental error is controlled with accuracy, the repetition on the central treatment is enough to obtain an appropriate estimation of the experimental error. On the contrary, in the field experimentation, it has been thought in the possibility of repeating the complete group of treatments with the aim of generating other error estimation.

Villasmil (13) mentions that due to the inconvenient of repeating the central treatment to obtain an uniform accuracy, other treatments designs were studied, as for example the San Cristóbal design and the orthogonalized San Cristóbal, until comparing these to the classic designs of the response surface methodology, this is how grows the necessity of generating a new design with more possibilities to be applied on the agricultural investigation, initiating the research of the called Orthogonal Central Composed design double star. This design surpassed the central orthogonal designs, rotary design and Orthogonalized San Cristóbal, using the efficiency criteria on the estimation of parameters of second order, while on the interaction it only surpassed the Orthogonalized San Cristóbal. It also generated the lowest variance of the estimated responses in combinations to the nucleus and generally, it can be considered as the D-optimum design among the compared designs.

Machado (7) made the comparison and practical application of the central design double star with a new star nucleus in fertilization essays. Later, Díaz (5) generated the response surface designs to estimate polynomies of third order: Plot designs 4k, Orthogonal Central Composed design and Composed Central Rotary design, using examples on each of the mentioned cases. Likewise, Castellanos (3) analyzed the behavior of experiments in divided plots with the main treatments arranged on a composed central rotary design, determining the probabilities of the mean squares for fixed and randomized effects. More recently, Guerrero (6), generated a new design called double composed central orthogonal design as an alternative to estimate polynomial models with two and three factors, which allows the use and study of 9 levels of each factor, facilitating a better study of the generated surface. This is a D-optimum design generating lower variances of the parameters estimators of the model and variances of the estimated response are the lowest when are near the center of the design, increasing at the time that it is getting far away from it.

This research aims to tackle the formal methodology for the analysis and interpretation of the designs composed of Box when fixed in incomplete blocks, expecting that it would constitutes a real alternative to face the variability problem that is generally presented on agriculture investigations on the field. The experimental error wants to be controlled in those cases that is relatively high the number of treatments to be essayed and group them in complete blocks would allow the inclusion of a higher level of heterogeneity. As well as for some cases of sequential experimentation, though when this technique is not yet too accepted on the field research, it can be eventually implemented on preliminary experiments and specially when a second order designs is assembled sequentially through one of first order and the performance of the complementary experiments required to construct the second order design, and during this time the test conditions between one and the other may change, being necessary the construction of blocks. The structure and efficiency of the Orthogonal Central Composed design and the Composed Central Rotary design are analyzed carefully in repeated incomplete and non repeated blocks.

The efficiency of the designs in incomplete blocks will be obtained through:

a) The variance of the experimental error.

b) The relative efficiency and

c) The variance of the regression coefficients.

Structure of the Composed Central design: Box and Wilson (1) introduce this design and mention their reasons of preference when they want to estimate the effect of second order. They say that the design consists on points of the vertex of a cube correspondent to a plot arrangement 2^k or any convenient fraction, with $(\pm1,\pm1,\bullet\bullet\bullet,\pm1)$, at the same time n_a "star" points are assigned with coordinates $(\pm\alpha,0,0,\bullet\bullet\bullet,0)$, $(0,\pm\alpha,0,0,\bullet\bullet\bullet,0)$,$\bullet\bullet\bullet$, $(0,0,0,\bullet\bullet\bullet,\pm\alpha)$ and n_0 points in the center with coordinates $(0,0,0,\ldots,0)$. The number of treatments on a central composed design, when using the nucleus S^k, is determined by the following expression: $N = 2^k + 2k + n_0$ where k = number of factors, = number of replications of the central treatments. Villasmil (8) says that the central treatment is repeated to obtain: a) the estimation of the experimental error, b) to reduce the variance of estimations, $v(\hat{y})$ in the center of the experimental region.

The geometrical scheme of the composed central design for k=3, includes eight points corresponding to the plot, belong to the vertex of the cube. The central point of the design corresponds to the center of the cube;

the six «start» points are at a distance of ± α of the point of the center of the design over the coordinated axis. In this research the selection of α to conferred the design will be done: a) Orthogonality b) Routability c) Orthogonality and routability simultaneously.

However, on the agricultural research and especially on essays done in the field, variability is very high and among other aspects, factors as weather, variations of the soil, genetic variability of the species and even the conduction of the essay make difficult the research. It is then reasonable that the estimation of the experimental error through the replications of the central treatment on the agricultural research could be seen carefully. Guerrero (6) says, that it is necessary to investigate designs alternatives that would allow to do a better estimation of the experimental error instead of doing it with replications of the central point and that the lack of the adjustment of the model might be measured adequately, this may be obtained with the replication of the total group of points of the design.

The block system on the designs composed of Box: The group of the vertex points of the cube and the group of axial points provide bases for a first division of the design composed in two blocks. This block will be orthogonal, if is possible to assign the central points to both parts, in a way that the total number of points per block be proportional to the amount of squares of each variable that contribute in the blocking.

If n_{c0} is the number of central points assigned to the block that contains the points of the cube and n_{a0} is the number of central points assigned to the block that contains star points, Box and Hunter (2) showed that for any composed design under the following expression:

$$\alpha = \left[\frac{n_c (n_a + n_{a0})}{2(n_c + n_{c0})} \right]^{1/2}$$

Orthogonality and routability is obtained in the blocking choosing $\alpha = n_c^{1/4}$. They also proved that in order to obtain an orthogonal blocking and routability simultaneously, it must fulfill:

$$\frac{(n_c)^{1/2}}{2} = \frac{n_c + n_{c0}}{n_a + n_{a0}}$$

It is important to mention that the group of star points can not be divided in sub-groups, since these, according to Box and Wilson (1), are rotary designs of first order. This subdivision is possible, nevertheless, for the group of points of the vertex of the cube, thus generating a confusion system for the plots designs 2^k in such a way that all the confused comparisons correspond to interactions of a higher order. If this is real, the confused comparisons will not associate with comparisons used to estimate the polynomies coefficients. This will depend if the cube is divided in sub-groups each one containing the same number of points, therefore, according to the expression to obtain the orthogonality previously mentioned, an equal number of central points must be added to each subgroup in order to keep the orthogonality.

It must be mentioned that since the number of central points in any block must be an entire positive, it can not always reach the complete orthogonality or the exact routability among the quadratic variables and block. Box and Hunter (2) say that though of the extra work involved in the calculus, due to a light orthogonality, it is not too big and the efficiency lost is negligible, in the practice designs can be used where the effects of the block are completely orthogonal scarifying a little bit the routability condition.

Replication in the designs composed of Box.

Box and Wilson (1), conceive the composed designs to give solutions to the experiment problems that involve a number of factors and/or relatively high levels and as Martínez Garza says (9), it highest application has taken place on industrial investigations where conditions allow an adequate control of the experimental error. Chacín (4), says that these mistakes ca be considered low, if are compared to the errors that can be obtained on the agricultural investigation, specifically on the filed experimentation where it is not always possible to keep throughout the experiment an uniform control of the experimental error, contrary to what happen on the industrial processes.

He also says, that as a consequence to the latter the way of measuring the experimental error in the industrial processes is only by repeating the central treatments. However, it has been proved in research done by Villasmil (8), that on the agriculture investigation, to work with only one replication of the basic nucleus of these designs may over-estimate the real randomized variability of the field, therefore causing a non real information; they mention that the inappropriate estimation of the experimental error is because it is only done with the replications of the central treatment in a same block.

With all these emerges the alternative of doing replications of the Basic nucleus in order to introduce a new source of error due to the interaction of treatments by blocks and compare it to the error, due to the central treatments with the aim of obtaining an appropriate measurement of the randomized variability.

Materials and methods

With the aim of illustrating the practical application of the methodology, information coming from a maize essay was used, where the effect of nutriments nitrogen, phosphorus and potassium were evaluated on the yield, and it was carried out on the Experimental station of Santa Cruz of Aragua.

Location: The essay was done

on the Experimental Station of Irrigation at Santa Cruz of Aragua, Sucre district, Aragua state, Venezuela. Location: 10º11’ of latitude and 67º 30’ of longitude and at a height of 444 msnm. Annual mean precipitation: 956.19 mm (a ten-year-period 1970/ 1979). Monthly mean temperature: 25.1ºC. Edaphological conditions: soils have a paedogenetic development formed through alluvial sediments. The most common textures are slimy clayey loamy and clayey. The retention capacity of humidity is high. There is low permeability and a mild restricted infiltration. Soils imperfectly drained with a sub-angular block structure. Sow: Sow was done using an Obregón hybrid, and was harvested within 127 days. 30 squared meters of treatment were used, sowing as effective plot 10 squared meters of treatment. Design: The used designs were: Composed Central Rotary, Orthogonal Central Composed, Orthogonalized San Cristóbal and plot design 33. These designs were «superimposed», this according to Chacín (4) refers to: «the response surface designs generally have common treatments, therefore, experiments can be done with treatments that are used either in one or other design, and are located on a randomized plots design that at the same time will allow to compare experimental errors obtained with replications in all the treatment group or by using the design with replication». In this experiment, treatments corresponding to the Orthogonal Central Composed design and Composed Central Rotary design were arranged in two complete blocks (replication one and replication two). During the development of this example, randomized completed blocks are designated as replications reserving the term block, for the divisions of the total grouping of treatments inside each replication (incomplete blocks). Considering the information correspondent to replication 1 of the Orthogonal Central Composed design, this replication is a complete block originating a total of 20 experimental points. On this example, it is supposed that these 20 points were assigned at two incomplete blocks, in block number one were put 2k + nco treatments; that is: (-1-1-1, 1-1-1, -11-1, 11-1, -1-11, 1-11, -111, 111, 000, 000, 000) and in block number two $2k + n_{\alpha o}$ treatments , that is: (-α00, α00, 0-α0, 0α0, 00-α, 00α, 000, 000, 000). In short, every replication was divided in two orthogonal blocks, where one was assigned the points correspondent to the plot arrangement and to the rest, the star points. Central points were spread between both blocks in the proportion that it would guarantee orthogonality or routability according to the case.

On the orthogonal design the value of α in 2k axial points is 1.81, different to 1.524 correspondent to the original information, and therefore it is also different the response in these points, the reason for these is as how it has been mentioned, the construction of blocks, for either the Orthogonal Central Composed design as well for the rotary, this implies the calculus of α that would allow the expected orthogonality among the effects of the treatment and the blocks effects, hence, the response of each of the points that include these new values of a, was estimated using the regression models generated with the original information. This fact does not affect the measurement of the blocks effect, since this is exclusively obtained through the response in the central points, though it must be said that it does affect the estimation.

Incomplete Repeated blocks in Composed Designs of Box.

A design alternative wants to be created that would be consistent in repeating the total group of points in a central composed design, which have been previously arranged in orthogonal blocks, so inside each complete replication there is an incomplete repeated block. This proposal is done under the belief that with this arrangement a better control of the high variability that involves the experiment in the field would be obtained, consequently, a better estimation of the experimental error. In other words, it is thought that the replication of the total group of treatments is not enough, but additionally these must be arranged in blocks. The analysis methodology is similar to the repeated central composed design in a sense that two error sources are generated, which must be compared with the aim of defining which one will be used on the hypothesis tests for effects of the treatment, or if it is valid to group them obtaining only one error. Regarding the estimation of parameters, this does not suffer any modifications.

For the achievement of the mentioned objectives, the attention was centered on the variance analysis results. These are presented as: a) Analysis of each replication separately and b) Group analysis of both replications, in the blocking as well as without it, and for each of the designs (orthogonal and rotary). In all cases, the error of designs with blocks compare to the error of designs without blocks calculating the relative efficiency, and the estimation of parameters and their corresponding variances are presented for all models, finally, the designs in incomplete blocks to those corresponding to complete blocks in terms of the relative efficiency of estimations are compared according to the following expression: $E_{1:2}=[v(\beta_i)2*N_2]/[v(\beta_i)1*N_1]$ Myers (10) cited by Chacín (4).

Results and discussion

Table 1 shows the scheme of variance analysis corresp-onding to the standard central composed design where b_0, b_i, b_{ii} y b_{ij} are the estimations of the parameters.

For the specific case of the rotary central composed design, the analysis is similar to the one showed, excepting by the fact that quadratic terms do not detach due to they are not estimated independently. It can be seen in table 2 the reduction of the

Table 1. Variance analysis scheme for the standard composed central design.

F of V	g. l.	SC
Linear terms	k	$\sum_{i=1}^{k} b_i (iy)$
X_i	1	k
Quadratic terms	k	$b_0 (0y) + \sum_{i=1}^{k} b_{ii} (iiy) - \left(\sum_{u=1}^{N} y_u \right)^2 \Big/ N$
X_{ii}	1	$b_0 (0y) + b_{ii} (iiy) - \left(\sum_{u=1}^{N} y_u \right)^2 \Big/ N$
Crossed terms	$\frac{k(k-l)}{2}$	$\sum_{i=1}^{K} \sum_{j=1}^{K} b_{ij} (ijy)$ $i < j$
$X_i X_j$	1	$b_{ij} (ijy)$ $i < j$
Lack of adjustment	$N - r_0 \frac{k(k+3)}{2}$	SC Total - SC regresión - SC Error
Error (T. C.)	$n_0 - 1$	$\sum_{u=1}^{n_0} y_{0u}^2 - \left(\sum_{u=1}^{n_0} y_{0u} \right)^2 \Big/ n_0$
Total	N -1	$\sum_{u=1}^{N} y_u^2 - \left(\sum_{u=1}^{N} y_u \right)^2 \Big/ N$

liberty degree of error due to the inclusion of blocks as a new source of variation. It must be said, that central treatments in tables 3 and 4 are not considered replications unless they are in the same block, therefore the amount of the error squares must be calculated considering this situation.

In tables from 5 to 8 can be observed that all variation coefficients are inside the appropriate rank (10-

Table 2. Variance analysis scheme for the central composed design in incomplete blocks.

F of V	g. l.	SC
Linear terms	k	$\sum_{i=1}^{k} b_i(iy)$
X_i	1	$b_i\,(iy)$
Quadratic terms	k	$b_0(0y)+\sum_{i=1}^{k} b_{ii}(iiy)-\left(\sum_{u=1}^{N} y_u\right)^2 / N$
X_{ii}	1	$b_0(0y)+b_{ii}(iiy)-\left(\sum_{u=1}^{N} y_u\right)^2 / N$
Crossed terms	$\frac{k(k-1)}{2}$	$\sum_{i=1}^{K}\sum_{j=1}^{K} b_{ij}(ijy)$ $i<j$
$X_i X_j$	1	$b_{ij}(ijy)$ $i<j$
Lack of adjustment	$N-r_0\frac{k(k+3)}{2}$	SC Total - SC regresión – SC Error - SC bloques
Block	m -1	$\sum_{w=1}^{m} B_w^2 / n_w - \left(\sum_{u=1}^{n_0} y_{0u}\right)^2 / n_0$
Error (T. C.)	n_0 - 2	$\sum_{u=1}^{n_0} y_{0u}^2 - \left(\sum_{w=1}^{m} B_w^2\right)^2 / n_w$
Total	N -1	$\sum_{u=1}^{N} y_u^2 - \left(\sum_{u=1}^{N} y_u\right)^2 / N$

Table 3. Variance analysis scheme for the completely repeated composed central design.

F of V	g. l.	SC
Linear terms	k	$\sum_{i=1}^{k} b_i(iy)$
X_i	1	$b_i(iy)$
Quadratic terms	k	$b_0(0y)+\sum_{i=1}^{k} b_{ii}(iiy)-\left(\sum_{u=1}^{N} y_u\right)^2 \Big/ N$
X_{ii}	1	k
Crossed terms	$\frac{k(k-l)}{2}$	$\sum_{i=1}^{K}\sum_{j=1}^{K} b_{ij}(ijy)$ $i<j$
$X_i X_j$	1	$b_{ij}(ijy)$ $i<j$
Lack of adjustment	$N-r_0\frac{k(k+3)}{2}$	SC Total - SC regressionn - SC Error- SC blocks
REP	r -1	$\sum_{u=1}^{N} y_u^2 \Big/ T - Fc$
Error (T. C.)	$r(n_0 -1)$	$\sum_{u=1}^{n_0} y_{0u}^2 - \left(\sum_{w=1}^{m} B_w^2\right)^2 \Big/ n_w$
TRAT*REP	(r –1)(T –1)	SC Tot. – SC Trat. – SC Rep.
Total	rN -1	$\sum_{u=1}^{N}\sum_{j=1}^{r} y_{uj}^2 - Fc$

Table 4. Variance analysis scheme for the central composed design in repeated incomplete blocks.

F of V	g. l.	SC
Linear terms	k	$\sum_{i=1}^{k} b_i\left(iy\right)$
X_i	1	$b_i(iy)$
Quadratic	k	$b_0\left(0y\right)+\sum_{i=1}^{k} b_{ii}\left(iiy\right)-\left(\sum_{u=1}^{N} y_u\right)^2 \Big/ N$
X_{ii}	1	K
Crossed terms	$\frac{k\left(k-l\right)}{2}$	$\sum_{i=1}^{K}\sum_{j=1}^{K} b_{ij}\left(ijy\right)$ $i<j$
X_iX_j	1	$b_{ij}\left(ijy\right)$ $i<j$
Lack of adjustment	$N-r_0\,\frac{k\left(k+3\right)}{2}$	SC Total - SC Regression – SC Error
Blockss/REP	r(m –1)	$\sum_{l=1}^{r}\sum_{w=1}^{m} B_w^2 \Big/ n_w - \left(\sum_{l=1}^{r}\sum_{w=1}^{m} B_w\right)^2 \Big/ mr$
REP	r -1	$\sum_{u=1}^{N} y_u^2 \Big/ T - Fc$
Error (T. C.)	$r(n_0$ -2)	$\sum_{u=1}^{n_0}\sum_{k=1}^{r} y_{0uk}^2 - \sum_{k=1}^{r}\left(\left(\sum_{u=1}^{n_{ob}} y_{ou}\right)^2 \Big/ n_{ok}\right)$
TRAT*REP	(r –1)(T –1)	SC Tot. – SC Trat. – SC Rep.
Total	rN -1	$\sum_{u=1}^{N}\sum_{j=1}^{r} y_{uj}^2 - Fc$

Table 5. Statistical analysis for the orthogonal design

REP	R^2	C.V.	Root CME	Y Medium
1	0.593	15.041	0.576	3.830
2	0.469	13.714	0.513	3.740
1 y 2 *	0.796	13.706	0.518	3.785

Table 6. Statistical analysis for the rotary design

REP	R^2	C.V.	Root CME	Y Medium
1	0.751	12.429	0.486	3.914
2	0.419	16.049	0.592	3.688
1 y 2 *	0.832	13.650	0.518	3.801

Table 7. Statistical design for the orthogonal design in incomplete blocks

REP	R^2	C.V.	Root CME	Y Medium
1	0.960	17.270	0.651	3.773
2	0.924	11.955	0.425	3.558
1 y 2 **	0.954	15.011	0.550	3.666

Table 8. Statistical analysis for the rotary design in incomplete blocks.

REP	R^2	C.V.	Root CME	Y Medium
1	0.975	12.182	0.489	4.017
2	0.974	8.776	0.298	3.405
1 y 2 **	0.976	10.926	0.405	3.711

20%) and that there is an important increment in the determination of models when incomplete blocks are used. In cases named as *the comparison of the error due to the interaction of treatments with blocks and the error product of replications of the central treatment through the F test, allowed establishing that both are estimators of the same variance and it was possible to group them in order to obtain a same error. On the contrary, in cases named by **, the F test shows that there are differences and the error correspondent to the replications of the central treatment is lower, therefore, in this error there might be an over-estimation of the existent real variability.

Comparisons of designs with the relative efficiency.

In the following tables, designs are compared in terms of mean squares of their experimental errors, for this purpose in all cases the efficiency of the design with block was measured compare to the one without blocks, according to the following expression:

$$ER = \frac{(n_1+1)(n_2+3)S_2^2}{(n_2+1)(n_1+3)S_1^2}$$

This expression compares the efficiency of the design 1 compare to design 2 being:

n_1 = liberty degrees of the error corresponding to the design with block

n_2 = Liberty degrees of the error corresponding to the design without block

S_1^2 = mean square of the error of the design with blocks

S_2^2 = Mean square of the error of the design without blocks.

The design with blocks will be more efficient than the design without blocks if ER is higher than 1. When the replication designs of the basic core of treatments are compared, can be observed in these that only for the rotary the design alternative in orthogonal blocks resulted to be more efficient than the corresponding cases without blocks, additionally, only in replication one of the orthogonal, the medium square of the error for the specific case with blocks is inferior than the corresponding without blocks (tables from 9 to 12). These results are considered a favorable response in terms of the objectives of this research.

In tables 13 and 14, the blocking alternative is compared compare to non-blocking in terms of the error sources when the basic nucleus of the composed design is repeated, that is, analyzing replications 1 and 2, the orthogonal and the rotary respectively. Here, the blocking alternative is not translated in higher efficiency. It is observed that in almost all cases, medium squares of errors for the designs with blockings are higher than those without. On this matter, it can be said that the basic nucleus of treatments that constitutes the composed designs, originates a better estimation of the randomized variability of essays carried out in the field, where it is already known that it is high.

Tables 15 and 16 compare the designs in incomplete blocks to those without blocking. When replications are evaluated separately, only in replication one of the rotary central composed design is showed a better efficiency of the incomplete block, however, for the repeated designs, it is observed a clear superiority of the incomplete block

For the case of the rotary central composed, the highest efficiency in the estimation of coefficients was more marked, even being more than 100% efficient, in all terms of the model. It is thought with these results that the incomplete blocking is not enough, but it is also needed to repeat the total group of treatments.

Table 9. Error variance and relative efficiency for the non repeated "orthogonal" composed central design (replication one).

	Replication one			
Structure	Source	gl	Error	Efficiency
Without	T. central	5	0.349380	0.783370430
Blocking	T. central	4	0.424758	

Table 10. Error variance and relative efficiency for the non repeated "orthogonal" composed central design (replication two).

	Replication two			
Structure	Source	gl	Error	Efficiency
Without	T. central	5	0.189141	0.995134578
Blocking	T. central	4	0.181015	

Table 11. Error variance and relative efficiency for the non repeated "rotary" composed central design (replication one).

	Replication one			
Structure	Source	gl	Error	Efficiency
Without	T. central	5	0.349380	1.388795310
Blocking	T. central	4	0.239591	

Table 12. Error variance and relative efficiency for the non repeated "rotary" composed central design (replication two).

	Replication two			
Structure	Source	gl	Error	Efficiency
Without	T. central	5	0.189141	2.01679314
Blocking	T. central	4	0.089317	

Table 13. Error variance and relative efficiency for the repeated "orthogonal" composed central design.

Source	Without	gl	Blocking	gl	Efficiency
Error (T. Central)	0.269260	10	0.30288685	8	0.859591107
Error (Trat. * Rep)	0.271097	14	0.86545044	14	0.313243817
Total Error	0.540357	24	1.168337729	22	0.499500927

Table 14. Error variance and relative efficiency for the repeated "rotary" composed central design.

Source	Without	gl	Blocking	gl	Efficiency
Error (T. Central)	0.269260	10	0.16445439	8	1.583167484
Error (Trat. * Rep)	0.270225	14	0.70663637	14	0.382410262
Total Error	0.539485	24	0.87109076	22	0.615357573

Table 15. Relative efficiency in terms of the estimator's variance of the incomplete blocks vs. the estimator's variance in complete blocks (DCCO).

effect	Rep. 1	Rep. 2	Rep. 1 y 2
N	0.624033	0.837134	1.722352
P	0.624033	0.837134	1.722352
K	0.624033	0.837134	1.722352
N*N	0.429365	0.575951	1.184961
P*N	0.718186	0.963391	1.982104
P*P	0.429365	0.575951	1.184961
K*N	0.718186	0.963391	1.982104
K*P	0.718186	0.963391	1.982104
K*K	0.429365	0.575951	1.184961

Table 16. Relative efficiency in terms of the estimator's variance of incomplete blocks vs. the estimator's variance in complete blocks (DCCR).

Effect	Rep. 1	Rep. 2	Rep. 1 y 2
N	1.457207	0.71545	2.086764
P	1.457207	0.71545	2.086764
K	1.457207	0.71545	2.086764
N*N	1.128563	0.999346	2.333655
P*N	1.422559	0.698426	2.037126
P*P	1.552901	0.762425	2.223757
K*N	1.034205	0.915457	2.127784
K*P	1.422559	0.698426	2.037126
K*K	1.552901	0.762425	2.223757

Conclusions

According to the results obtained in this research, the replication of the total group of treatments in the designs composed with Box, it is convenient to obtain estimations of the most efficient and logical coefficients, as well as variances in harmony to the real variability of the agriculture field, corroborating the results obtained by other researchers.

From the theoretical point of view and according to the results of the illustrative example, is it thought that the proposed alternative that consists on the replication of the basic nucleus of treatments with the incomplete orthogonal blocking system inside each replication is convenient and favorable in the agricultural trial.

In the case of the repeated orthogonal central design; incomplete blocking was from 18 to 100% more efficient compare to the same design without incomplete blocks. In the Rotary central composed design, the incomplete blocking resulted to be 100% more efficient in all cases.

The determination of models was more superior in the designs where there was an incomplete blocking, in relation to those where there was not. The incomplete blocking improves the adjustment of models.

The statistical analysis for the designs composed with Box in repeated incomplete blocks proposed in this research, allows measuring efficiently the local control that is done with the incomplete blocking and it also allows selecting the most adequate experimental error for the hypothesis tests of the coefficients of the model.

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